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A New Approach for the Dynamic Modelling of Credit Risk

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Abstract

It is well known that static models for credit risk fail to meet the increasing demand for hedging credit derivatives since they fail to track the credit risk change of a structured portfolio over multiple time periods. In this paper, we propose a dynamic credit risk model based on asset growth rate. The model can be used to dynamically analyse and price mainstream credit derivatives, is easy to calibrate, and captures well both bullish and bearish credit market conditions. We provide two alternative candidates for default conditions and we evaluate them. We illustrate our model for a CDO-type contract. As a dynamic structural model, our approach does not rely on certain type of distributions. However, further extensions can be made to assess the effects of exogenous factors such as pairwise correlation and interest rate. Following the recent market slide and the ongoing credit crunch, we believe that an "old school"-type intuiyive approach will be valuable for market practitioners who are increasingly focusing on new routes to mitigate and hedge risk exposure.

JEL Classifications: G01, G13, G17.

Keywords: Dynamic credit risk models, default-able bond prices, multi-step Monte Carlo simulations.

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1 Introduction

The study of credit risk and the valuation/hedging of credit derivatives are one of the most popular and controversial issues that concern the entire financial industry. Increases of defaults and bankruptcies during the recent credit crunch has stipulated a heated debate about the adequacy of the existing pricing and hedging methodologies for portfolios of credit derivatives.

The main objective of this work is to propose and evaluate a treatable dynamic framework that addresses many of the deficiencies of the standard market model.

The structure of the paper is as follows. Section 2 provides an overview of the current mainstream approaches in the credit risk market. Section 3 describes our main contribution, that is a dynamic credit risk model based on asset growth rate. Section 4 illustrates the implemantation and calibration of our framework on a Collateralised Debt Obligation (CDO) contract and discusses possible extensions of our work. Finally, Section 5 concludes the paper.

2 Overview of Credit Models

We start by reviewing current mainstream approaches in the credit risk market.

2.1 Static Copula Models

The current standard model, also known as the Gaussian copula model, starts with a simple one-factor specification of the change in the asset value of the reference company and in turn, determines the firm's time to default.

This approach was originally introduced by Vasicek (1991), Li (2000) and recently developed by Laurent and Gregory (2005). In esence, the default probability over the whole life of the contract is determined by the normally distributed asset value, in which case, if the asset value is high then the probability of default is low, and verse versa. *Default* is defined as the first time the asset value falls down cross a predefined value barrier.[1]

The model assumes a constant hazard rate and ignores the change of probability of default over the whole time period. Considering only the loss distribution, many alternative copula and distribution functions to Gaussian copula have been suggested,[2]

[1] Normally the debt value of the company, or the face value in case of a bond.

[2] See Hull and White (2004), Burtschell, Gregory, and Laurent (2009), Guegan and Houdain (2005), and Kalemanova, Schmid and Werner (2007) for detailed discussions on copula functions.

including: the student t-copula, the double t-copula, the Archimedian copula, the Clayton copula, the Marshall Olkin copula, and distributions like Normal Inverse Gaussian and Variance Gamma.

This type of model is, in general, static due to not able to describe how the default environment evolves. However, the following two insights are worth keeping:

1. The probability of default (PD), which is conditioned on a market momentum of Y, is related to the default barrier K with a normal distribution function:

$$p(Y) = \Phi\left(\frac{K - \rho \cdot Y}{\sqrt{1-\rho^2}}\right). \tag{1}$$

2. The relationship between the default probability and the asset value is expressed as:

$$A = \Phi^{-1}(1-p). \tag{2}$$

2.2 Prior Dynamic Models

In order to fit and analyse data covering multiple time horizons, and for more accurate valuation of derivatives on credit related financial contracts, dynamic models were introduced. We first provide a quick review of such attempts before looking into the details of a specific model in the next subsection.

Structural Models

This approach was origined by Black and Scholes (1973) and Merton (1973), and recently extended by Albanese *et al.* (2006), Hull, Predescu and White (2009), and Baxter (2006). The basic model is similar to the Gaussian copula model. Correlated asset values are constructed by a univariate drift factor and an idiosyncratic stochastic process for every asset.

Anson *et al.* (2004) have shown that most research on credit derivatives with a structural type of model analyses the instrument like an option. And default is specified as when the value of the asset crosses a predefined barrier.

Structural models have the advantage that they are based on market observables and have sound economic underpinnings. But their main drawback is that they are generally hard to calibrate and computational expensive.

Reduced-form Models

This type of models (also called the intensity models) focuses on modeling the correlated evolving processes for the default probability of the referenced companies. Pioneers of this approach include Jarrow and Turnbull (1995), Lando (1998), and Duffie and Singleton (1999).

In general, it is hard for reduced form models to fit market data well because of their weak link to market variables. Most importantly, the widely used Poisson process is a counting process which means that the counter increases as time goes by, in this case, the variance of correlation and default probability is limited. Hence, many researchers add jumps in the hazard rate process to describe the 'surprises', examples can be found in Zhou (1997), Duffie and Singleton (1999), and Duffie and Garleanu (2001).

Most models consist of a self-evolving component, a market contingent component and/or an industry contingent component. The Duffie and Garleanu (2001) model, for example, has all three, and each of the components follows a process led by a diffusion and a jump factor.

Recent extensions and models of this approach can be found in Graziano and Rogers (2009), Hull and White (2008), Chapovsky, Rennie and Tavares (2007), and Hurd and Kuznetsov (2006).

Stochastic Loss Distribution Approach

This methodology was first originated by Heath, Jarrow and Morton (1992), Jarrow and Turnbull (1995), and Jarrow, Lando and Turnbull (1997). Distinguishing this type of models as an individual category is controversial[3] because these frameworks focus on the probability of the losses of portfolio to take place or reach some level in the future. Thus, this approach is also referred to as the "top down" approach.

Recent research includes an extension of Heath, Jarrow and Morton (1992) with a loss deduction assumption in Sidenius, Piterbarg and Andersen (2008). Bennani (2005) assumed that the instantaneous loss is a percentage of the remaining principal. Errais, Giesecke and Goldberg (2006) suggest a model of default probability with jumps, while Longstaff and Rajan (2006) suggest that it is the loss that follows a jump process and different types of jumps are tested.

Meanwhile, Markov chains are widely adopted in this literature. Schonbucher (2006) and Walker (2009) considered the evolution of loss distribution in a Markov loss model.

[3] See Anson *et al.* (2004) and Choudhry (2005).

2.3 Duffie-Singleton Discount Rate Approach

Duffie and Singleton (1999) proposed a Reduced Form model characterizing the default exogenously by a jump process. The event of default is led by a hazard rate and the losses at default is parameterized as a fractional reduction in pre-default market value.

Suppose we have a corporate bond paying X at maturity time T, denote the hazard rate at time $0 \leq t \leq T$ by h_t and the expected fractional loss at time t by L_t. Under a risk-neutral environment, $h_t L_t$ stands for the 'mean-loss rate', thus if the risk free interest rate r is replaced by a adjusted short rate R, where $R = r + h_t L_t$, the market value of this bond at time zero is then given by

$$V_0 = E_0^Q \left[e^{-\int_0^T R_t dt} X \right], \tag{3}$$

where Q is the risk-neutral martingale measure.

As the mean-loss rate $h_t L_t$ does not depend on the bond value, if R is chosen carefully, standard term-structurer default-free debt models are directly applicable to default-able debts by replacing the risk-free rate r by R.

Under this set up, let us denote the unit recovery at time $t+1$ by φ_{t+1}. It is natural that the contract value at time t consists of two parts: (i) $h_t \cdot e^{-rt} \cdot E_t^Q(\varphi_{t+1})$ for the event of default and (ii) $(1-h_t) \cdot e^{-rt} \cdot E_t^Q(V_{t+1})$ as the bond value continuous to evolve at time $t+1$ in case of no default. Mathematically, the bond value at time t is expressed as

$$V_t = h_t e^{-rt} E_t^Q(\varphi_{t+1}) + (1-h_t) e^{-rt} E_t^Q(V_{t+1}). \tag{4}$$

Meanwhile, as the unit recovery of market value at time $t+1$ is the difference between real market value at time $t+1$ and the expected fractional loss at time t, i.e.

$$E_t^Q(\varphi_{t+1}) = (1 - L_t) E_t^Q(V_{t+1}). \tag{5}$$

Substituting equation (5) into equation (4), we can rewrite equation (4) as

$$V_t = (1 - h_t L_t) e^{-rt} E_t^Q(V_{t+1}) \tag{6}$$

The default adjusted discount factor e^{-Rt} at time t is given by

$$e^{-Rt} = (1 - h_t L_t) e^{-rt}. \tag{7}$$

Using the well known result that for a small number ϵ, e^{ϵ} is approximately equal to

$1+\epsilon$, similarly if the contract time is observed in small length, we can approximately have: $R_t \cong r_t + h_t L_t$. Now if one recursively solves equation (6) for the whole time interval, it is easy to have:

$$V_t = E_t^Q(e^{-\sum_{i=t}^{T-1} R_i} X). \tag{8}$$

And thus the financial contract is priced.

The authors then derived fair prices of securities which are prone to default risk. In summary, this approach provides a default-able version of Heath, Jarrow and Morton (1992) model. The authors concluded that their framework is not suitable when dealing with non-callable bonds, because h_t and L_t must work together as the 'mean-loss rate' in this model and cannot be identified separately from data of default-able bond prices alone.

We follow the Duffie and Singleton (1999) approach and take this work as a precursor of our framework, as a main advantage of this approach is that it is possible to directly calibrate model variables to observable market prices such as corporate bonds. Further, it is possible to parameterize R directly as it is exogenous. However, the downside of this type of model is, as the loss is priced with a default adjusted 'risk free' rate, the final value is expressed as an exponential function and thus the model is not suitable for pricing financial contracts like CDS which has no payoff at maturity.

3 Dynamic Growth Rate Model

In this section, we propose a dynamic credit risk model based on asset growth rate. The model can be used to analyse and price all mainstream credit derivatives overtime. Further, it is easy to calibrate and captures well both bullish and bearish market movements. Finally, this growth rate model considers the value evolving process forwardly from time 0, thus only an initial condition is needed and zero face-value instruments are covered as well. We provide two approaches of default conditions for the model and a detailed robust test is given in later sections.

3.1 Notations

Unless specified, we use the following notations for the remainder of this paper.

T : contract maturity.

$0 \le t \le T$: a general time before maturity.

N : number of referenced companies.

$0 \leq i \leq N$: i^{th} company in the portfolio.

$n(t)$: number of cumulative defaults at time t.

L_n^t : cumulative loss on the portfolio at time t.

r : risk-free interest rate.

R_i : default-adjusted asset growth rate for company i.

c_i : coupon rate of company i. Also defined as c for all underlying bonds in a homogeneous portfolio.

e^{Rt} : default-adjusted unit asset value at time t.

$p(t)_i$: default probability of company i at time t.

τ_i : time of default for company i.

s : premium spread.

$A(t)_i$: asset value of company i at time t.

K_i : default threshold for company i.

$x(t)_i$: stochastic short growth rate of company i at time t.

3.2 Model Setup

Suppose we have a portfolio of default-able zero coupon bonds and assume that these constitute the only debt of the referenced names. Furthermore, assume that the portfolio is homogeneous, each bond accrues an interest rate of $r+c$, which means the discrete corresponding asset growth factor over a short time interval is: $e^{(r+c)}$.

As the asset value evolves until maturity T, the bond erases its market value if its (time t) growth rate is lower than the risk-free *Treasury Bill* interest rate. In other words, the bond is down graded to junk in this case, and a credit event is triggered.

We assume the growth rate of a referenced company consists of two components: the risk-free rate r and a stochastic short growth rate x. The latter follows an Ornstein-Uhlenbeck type process:

$$dx = -axdt + \sigma dz, \tag{9}$$

where a is the drift and σ is volatility, z is a Brownian Motion. Solving equation (9) we have:

$$x(t) = x(0)e^{-at} + \sigma \int_0^t e^{-a(t-u)} dw_u. \tag{10}$$

Assume that the time zero value of the underlying bond is 1, thus the asset value growth for time interval $(0, t)$ given by the growth rate is: $e^{(r+c+x(t))t}$, where $x(t)$ is given by the equation above. Then the value of default can be calculated and thus the probability of default at each time t before maturity is found.

We illustrate how to find $p(t)$ with a simple numerical example: Say that the yield of a risk-free, five-year, zero coupon Treasury bond paying £100 at maturity is 3%. Also, the yield of a zero coupon, zero recovery corporate bond with the same face value and maturity is 4%. Then, at present, the Treasury bond worths: £$100e^{-0.03\times5} = 86.071$ and the corporate £$100e^{-0.04\times5} = 81.873$. The value of default is their difference, £4.198.

In case of default, the corporate bond will cause a loss of full face value of £100 at maturity, so the risk-neutral expected loss from this default is simply: $100e^{-0.03\times5}p(0)$. Hence,

$$100e^{-0.03\times5}p(0) = 100e^{-0.03\times5} - 100e^{-0.04\times5},$$

and thus

$$p(0) = \frac{e^{-0.03\times5} - e^{-0.04\times5}}{e^{-0.03\times5}}.$$

At the end of the first year, the value of the Treasury bond is increased with the risk-free rate 3% to: $\pounds 86.071 \times e^{0.03} = 88.692$. As for the corporate bond, if the growth of first year is lower than the promised 4%, say, 3.5% (this can be seen as an addition of the 3% risk-free rate and $x(1) = 0.5\%$), the value of the bond is now: $\pounds 81.873 \times e^{0.035} = 84.789$.

Ideally, one would expect the corporate bond to grow with an average rate of 4% every year during the five years and make it to the value of £100 at maturity, so the first year target would be: $\pounds 81.873 \times e^{0.04} = 85.214$. Thus, with the value of £84.789, it is more difficult to reach £100 and therefore a larger PD is obtained:

$$\begin{aligned}
100e^{-0.03\times4}p(1) &= 100e^{-0.03\times4} - 100e^{-0.04\times5}e^{0.035} \\
\Longrightarrow 88.692p(1) &= 88.692 - 84.789 \\
\Longrightarrow p(1) &= 0.044.
\end{aligned}$$

It follows that the probability of default is given by

$$p(t) = \frac{e^{-r(T-t)} - e^{-(r+c)T+(rt+x(t))}}{e^{-r(T-t)}} = 1 - e^{x(t)-cT}. \tag{11}$$

It is obvious that at any generic time t, if $x(t) < ct$ the derived PD will be based on the performance of the bond during the first t years. On the contrary, if $x(t) > ct$, the corporate bond is doing well for the period $(0, t]$, and to obtain the probability for the rest of (t, T) years, one may simply focus on the promised yield $r + c$ and the maturity time. So the probability of default is now defined as

$$p(t) = \frac{e^{-r(T-t)} - e^{-(r+c)(T-t)}}{e^{-r(T-t)}} = 1 - e^{c(T-t)} \quad \text{for} \quad x(t) \geq ct.$$

Continue with our example, for any growth $x(1) \geq ct = 1\%$, by the end of the first year, we have the default probability of the corporate bond seen at time $t = 1$for the remaining $5 - 1 = 4$ years:

$$p(1) = \frac{88.692 - 85.214}{88.692} = 0.0392.$$

In short, the default probability with respect to the time dependent growth rate $x(t)$ is given by

$$p(t) = \left\{ \begin{array}{ll} x(t) \geq ct: & \frac{e^{-r(T-t)} - e^{-(r+c)(T-t)}}{e^{-r(T-t)}} = 1 - e^{c(T-t)} \\ x(t) < ct: & \frac{e^{-r(T-t)} - e^{-r(T-t)-cT+x(t)}}{e^{-r(T-t)}} = 1 - e^{x(t)-cT} \end{array} \right\}. \tag{12}$$

3.3 Trigger of Default

In this section we consider two alternative conditions of default. Subsequently, we compare our findings.

3.3.1 Growth Rate Factor

Having obtained the default-adjusted asset growth rate from the previous section, we observe that according to our assumptions, the asset growth is limited by a lower rate of e^{rt}, in other words, the variable $x(t)$ has to stay above 0 for the company to survive until time t. This implies that the unit value at time t is $e^{-(r+c)T+(rt+0)} = e^{-(r+c)T}e^{rt}$. Clearly, the default condition can be specified as: $x_i(t) > 0$ or $x(t) > 0$ for a homogeneous portfolio.

Continuing the numerical example of the previous section, the probability of default at $x\,(1) = 0$ is

$$p(1) = \frac{88.692\ -^{-0.03\times 4}\ -100e^{-0.04\times 5+0.03}}{88.692} = 0.0488.$$

For any $x\,(1) < 0$, the probability $p(1)$ becomes larger than 0.0488.

In this setting, a practitioner does not need to worry about the time dependent default probability given by equation (12), as the default is simply triggered when $x\,(t)$ falls below 0.

3.3.2 Asset Value Approach

As in Li (2000) and Kalemanova, Schmid and Werner (2007), the asset value approach implies that the default is triggered if the asset value falls below the threshold. The barrier is given by equation (2) using a static probability of default from a credit curve, i.e.

$$K = \Phi^{-1}(1 - p_k). \tag{13}$$

The variable p_k is defined as the probability of default over the whole time interval in the market model. Meanwhile the threshold K is normally considered as a constant for CDO type of contracts.[4] As for our example given above, p_k is obtained with $x\,(t) = 0$ where $p_k = 0.0488$, thus, $K = 1.6566$.

For the time dependent case, we can have a time t 'asset value' using equation (2) as

$$A(t) = \Phi^{-1}(1 - p(t)). \tag{14}$$

Use the same value of $x\,(1) = 0.5\%$ and $p(1) = 0.044$ that we had earlier in our example, $A(1) = 1.706$. So the asset survives until $A(t) < 1.706$.

In this case, a default is triggered if value $A(t)$ falls across the default barrier K, i.e. $A(t) < K$ or

$$\Phi^{-1}(1 - p(t)) < \Phi^{-1}(1 - p_k). \tag{15}$$

As the Φ^{-1} function decreases in value when $p\,(t)$ becomes higher, to hold true that $A(t) < K$ one will require $p\,(t) > p_k$, then apply equation (12) to obtain from equation (15) that $x\,(t) < 0$.

[4] For details see Anson *et al.* (2004), Bluhm and Overbeck (2007), and Loffler and Posch (2007).

3.3.3 Robust Test

Following the previous discussion, we summarise the two conditions of default in Exhibit 1 below:

Exhibit 1: Model calibration procedure of two approaches.

$$\underbrace{\text{Model engine generator of } x\,(t)}$$

$$\swarrow \qquad \searrow$$

$$\underbrace{\boxed{\text{The time } t\ PD,\ p(t),\ \text{and } p_k \text{ at barrier } K}} \qquad \boxed{\text{Default if } x(t) < 0}$$

$$\underbrace{\boxed{\begin{array}{c}\text{Asset value from survival function:}\\ A = \Phi^{-1}(1-p) \text{ with given } p(t) \text{ and } p_k\end{array}}}$$

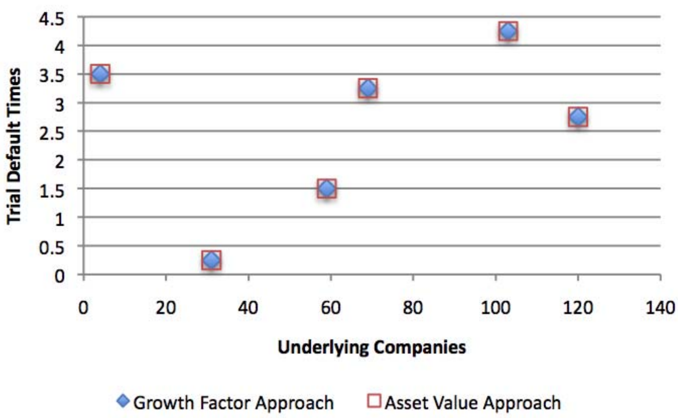
$$\boxed{\text{Default if } A(t) < A_k}$$

In general, the *Growth Factor* approach shown on the right-hand side of Exhibit 1 is easier to program and superior in computation time when simulating due to less calculations. Meanwhile, if the practitioner needs to observe the change in time-value and carry out portfolio or match to name hedging strategies based on asset value, the *Asset Value* approach is much more suitable because it works well with alternative asset value models. Based on the simulation results in hand one may easily plug in a different barrier and obtain the estimated defaults of the simulated data.

To examine the convergence and robust results on above discussed approaches, we apply them both with multi-step Monte Carlo simulation, using the same data set generated by the random variable generating engine, the simulated default is shown in the Figure 1.

Figure 1

Simulated Trial Default Times



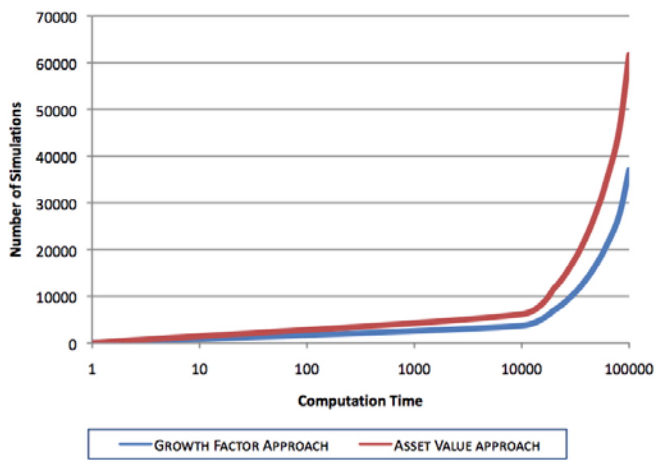
It is obvious that both approaches provide identical results, in other words, the same company defaults/survives at the same time in both approaches. Mathematically, we have:

$$\because A(t) < K \quad \text{i.e.} \quad \Phi^{-1}(1 - p(t)) < \Phi^{-1}(1 - p_k) \quad \therefore p(t) > p_k$$
$$\therefore \text{apply equation (11)} \quad 1 - e^{x(t)-cT} > 1 - e^{-cT} \quad \therefore x(t) < 0.$$

Figure 2 shows the computation time of the two default trigger approaches.[5]

Figure 2

Computation $\underset{\text{(seconds)}}{\text{Time}}$ vs. Number of Simulations



Clearly, the asset value approach is more computational expensive; however, this approach is more convenient when considering a time dependent underlying asset value. Thus the time cost is bearable when calculating hedging parameters of the underlying portfolio.

4 Model Implementation: Valuation of a CDO

In this section we implement and calibrate our model on a collateralized debt obligation (CDO) contract.

[5] The test is performed under Microsoft Excel VBA environment, the computer we used has Intel P4 3.6GHz CPU with 1G Memory.

4.1 The Simulation Procedure

In the occurrence of a credit event, we follow the Asset Value approach to produce the following steps in our simulation process.

1. Generate value $x(t)$ using equation (9) and (10) for each underlying company over the whole contract time.
2. Calculate the default adjusted asset value growth factor e^{Rt} for each time $0 \leq t \leq T$.
3. Calculate the time dependent default probability from equation (11) for all companies over the whole time period.
4. Calculate the expected time t asset value for all companies using the default probability from the above step.
5. Calculate the default barrier using equation (13).
6. Compare the expected asset value, the default barrier and determine credit trigger using equation (15). Defaulted companies are knocked out for the remaining contract life.
7. Calculate the cumulative loss from default. The recovery rate is chosen to be 40% inline with market standards.
8. For each tranche, the loss at each time is given by the comparison between tranche size and the tranched loss. The fair spread is given by the component that equates the tranche loss with the tranche notional.
9. Repeat the above steps for a large number of times and calculate the average fair spread from all trials.

4.2 Numerical Results

The CDO-type contract we consider is the 5-year iTraxx Europe. Total underlying names is 125, and the six structured tranches are sized: 0-3%, 3-6%, 6-9%, 9-12%, 12-22%, and 22-100%. The payment days are set quarterly and the recovery rate is fixed to 40%.

Bearing in mind the on going tsunami in credit markets, we use two sets of market data, one bearish and a bullish. The first data set is the iTraxx Series5 which started on 20th September 2006 and ends on 20th December 2011. The market quote we use is recorded on the 31th of January of 2007,[6] with compound spread 23bps. The second data set is the latest on-the-run iTraxx Series8 version 1 with contract maturity the 20th December 2012. The quote date is 30st January 2008 with compound spread 123.75bps.

The interest rates from the Bank of England (BoE) are 5.5% and 5% respectively, and the σ factors (volatilities of the average CDO spreads in the datadase) are 0.0031 and 0.0072. The numerical results are summarised in Tables 1 and 2.

Table 1: Numerical results for iTraxx tranches on 31/1/2007.

Tranche	Market	Growth Rate
0% - 3%	10.34%	17.03%
3% - 6%	41.59 bps	64.36 bps
6% - 9%	11.95 bps	20.14 bps
9% - 12%	5.6 bps	2.7 bps
12% - 22%	2 bps	0.85 bps
Absolute Error		26.91 bps

Table 2: Numerical results for iTraxx tranches on 30/1/2008.

Tranche	Market	Growth Rate
0% - 3%	30.98%	37.03%
3% - 6%	316.9 bps	360.15 bps
6% - 9%	212.4 bps	247.31 bps
9% - 12%	140.0 bps	172.64 bps
12% - 22%	73.6 bps	85.28 bps
Absolute Error		22.48 bps

The bearish market simulation shown in Table 1 produce an almost 50% discrepancy to market data. Given that our absolute error is 26.91 bps,[7] we believe that the main reason for the large percentage differences is that for the 9%-12% and 12%-22% tranches the market spreads are low, thus the 1.15 bps error (2 bps-0.85 bps) for the 12%-22% tranche produces 57.5% error.

[6] All data is quoted from Markit and Reuters' CDS Views. Note that different contributors may submit different quotes.

[7] Note that the absolute error is the sum of the absolute difference between simulation outcome and market data excluding equity tranche, as according to market rules, equity tranches spread is locked to 500bps.

As for the intense post sub-prime market period, we can see from Table 2 that the simulated spreads are within a percentage difference of 15% compared to the market spreads. However, in tandem with Table 1, the differences are more pronounced for the last two tranches. The absolute error we observe here from each simulation outcome is about 5 times that of a good market, but if one considers the tranche spread level in 2008 compared with data from a year earlier being far more expensive (e.g. tranche 3%-6% spread is 8 times higher and the super senior tranche is 36 times more expensive), we reckon that our simulated results are still acceptable.

We may conclude that the growth rate model is useful in capturing both easy and intense market movements. Furthermore, our results have shown that the multi-step Monte Carlo simulation procedure is very productive when dealing with structural credit models.

4.3 Extensions

The assumptions in our simulation procedure were in accordance to the market standard. However, as the model is implemented with a multi-step simulation, 'richer' assumptions can also be added in case one needs to investigate a more complex market structure. Typical examples are given next.

4.3.1 Dynamic Recovery Rate

Although, the market practice is to set a uniform recovery rate R to all classes of credit derivatives, recent research explains the benefits of stochastic recovery rates in better fitting reality.

Yu (2003) and Herkommer (2007) have shown that the recovery rate can not be disassociated from default probability. Moreover, Hu and Perraudin (2002), and Altman *et al.* (2005) suggest that there is a negative correlation between the default probability and the recovery rate.

In our dynamic asset growth rate model, for each observed time t, a negative correlation between the derived default probability and the recovery rate is implied by our default conditions. Thus, our framework is compatible with recent empirical findings regarding the impact of the recovery rate.

4.3.2 Match to Name Correlation

According to reviews by Deacon (2003) and Anson *et al.* (2004) most distribution and copula based credit models are correlation centered, and as practitioners price the products with market standard Gaussian copula model, discussions on pairwise correlation between underlying companies will continue to be popular in the future.

To examine the effects of correlation factors within our model, we follow the approach proposed by Hull, Predescu and White (2009). The correlation parameter ρ is set to be embedded in the Brownian Motion process z, so we now have z as

$$dz_i = \rho_i dM + \sqrt{1-\rho_i^2} dZ_i. \tag{16}$$

Here i indicates the i^{th} company and M is a common Wiener process for all underlying names. Above equation adapt idiosyncratic correlation assumptions for each of the individual companies, so detailed correlation assumptions can be extended for more complex cases of correlation centered growth rates.

Thus, the growth rate process is driven by the macro market momentum together with an individual process. In this way, one may apply the match to name correlation factors of selected data with our proposed dynamic structure model and observe the change due to difference in correlations. Further, for time dependent simulation processes, our framework may well house the time dependent correlation assumption as the asset growth rate for each time step is distinguished from the previous steps.

4.3.3 Time Dependent Derivative Pricing

Since the 2007 credit crisis, the issue of hedging credit risk with other derivatives is heated more than ever before. As a dynamic model, our approach, can be used to perform continuous time cash flow analysis as well as valuation of option-type securities.

Early structural models such as Merton (1973) and Black and Cox (1976) consider credit default products as exotic options in which the default trigger condition is set as the strike price which makes an option exercisable. In their context, the asset value follows a log-normal process and the derivative is priced as follows. The well known solution of a vanilla call option is

$$C = A(0)\,\Phi(d_1) - Xe^{-rT}\Phi(d_1)\,,$$

where $A(0)$ is the asset value at time 0, X the option strike, r the risk-free interest rate and T the maturity, with d_1 and d_2 defined by

$$d_1 = \frac{\ln A(0) - \ln X + (r+\sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}.$$

Having the time dependent asset value $A(t)$, one is able to derive the time t call and/or put price for an option-type contract on credit default-able assets. Further, calculations of the *Greek* factors are straight forward and time dependent sensitivity analysis is made easy. Thus, time dependent hedging strategies can be developed on expected cash flow and sensitivity analysis from the *Greeks*.

5 Conclusions

In this paper we propose a new dynamic approach for structural credit risk modeling. We believe that a time dependent pricing technique is vital as the whole market is facing the challenge of actively managed and/or replicated credit portfolios. The growth rate model that we suggest is easy to calibrate as inputs are either given directly or easily derived from market data. It turns out that our framework can accommodate both bearish and bullish credit markets and fits market quotes reasonably well. Obviously, further extensions can be made to analyse the effects of many other factors which are exogenous to our model, such as the pairwise correlation and interest rates. Our philosophy though was to keep everything relatively simple and "old school", since in the recent climate market practitioners are increasingly returning to basics when exposed to credit risk.

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